DETERMINATION OF LOT SIZE IN THE CONSTRUCTION OF SIX SIGMA BASED SAMPLING PLANS

R. RADHAKRISHNAN[1] and P. VASANTHAMANI[2]

[1]Associate Professor, PSG College of Arts and Science, Coimbatore - 641 014, India.
rkrishnan_cbe@yahoo.com

[2]Associate Professor, SNR Sons College, Coimbatore - 641 006, India.

ABSTRACT

Six Sigma is a concept, a process, a measurement, a quality philosophy, and a management strategy used to improve the systematized quality of an organization, which when applied reduce the number of defects to 3.4 or less per million opportunities in the long run, a concept suggested by Motorola in 1980. The main aim of Six Sigma is to reduce costs/wastages as much as possible and improve the quality of the product to the maximum satisfaction of the consumer. Focusing on reduction of non-conformities will result in enhanced quality with more satisfaction and profit. The concept of Six Sigma can be applied in the process of quality control in general and Acceptance Sampling in particular. Various sampling plans have been designed focusing on the sample size only but not on the lot size. It is also unconvincing to say that the lot is accepted or rejected based on a fixed sample size irrespective of the huge lot size.

In this paper an attempt is made to determine the size of the lot (N) of a Six sigma based single sampling plan with Poisson distribution as a base line distribution. Tables are also constructed for the easy selection of the plan for various values of sample size (n) and Average Outgoing Quality (AOQ) with probability of acceptance $1 - 3.4 \times 10^{(-6)}$.

Key words—Six sigma, Single Sampling Plan (SSP), rectifying inspection, Sample size (n), Lot size (N), Lot quality (p), Average Outgoing Quality (AOQ).

1. INTRODUCTION

Six Sigma is the most popular tool to convert management problem to a statistical solution then convert it to a management solution. Six Sigma has a measurement of standard in product variation can be traced back to the 1920's when Walter Shewart showed that three sigma from the mean is the point where a process requires correction.

Rectifying inspection programs were among the earliest of sampling inspection schemes. The plans developed by Harold F. Dodge and Harry G. Romig (1942) at the Bell Telephone Laboratories were rectifying inspection plans. Later their ideas on rectifying inspection have been extended by G.A. Barnard and F.J. Anscombe (1949). Initially programs of rectifying inspection were associated with lot-by-lot acceptance sampling, but in 1943 Dodge also proposed a rectifying scheme for continuous production. Radhakrishnan and Sivakumaran (2008) constructed six sigma sampling plan indexed through Six Sigma Quality Level (SSQL) with Single Sampling Plan as a reference plan. All the plans suggested by them are for infinite lot size only.

A. Average Outgoing Quality (AOQ)

AOQ is the average quality of outgoing product including all accepted lots or batches plus all rejected lots or batches after the rejected lots or batches have been effectively 100 percent inspected and all defectives replaced by non – defectives.

B. Glossary *of symbols used*

P : Proportion defective / lot quality.

N : lot size

n : sample size

d : number of defectives counted

c : acceptance number

P_a : Probability of acceptance of the lot quality p

AOQ : Average Outgoing Quality

C. Operating Procedure

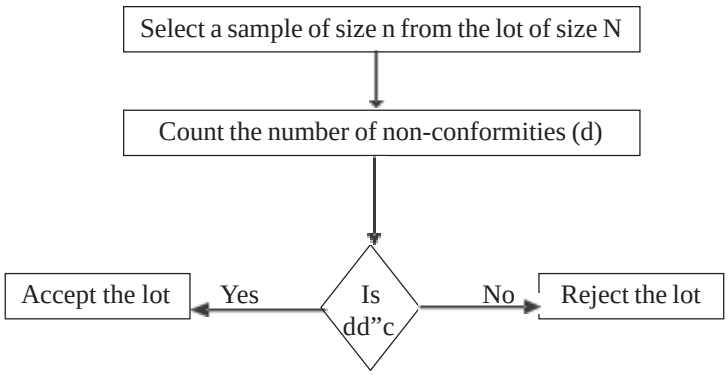
The operating procedure of a Single Sampling Plan (SSP) is given below with a flowchart 1:

Step 1 : Take a random of size n from the lot of size N and count the number of non-conformities (say d).

Step 2 : If d d" c, accept the lot.

Step 3 : If d > c, reject the lot.

Flowchart 1



D. Conditions for Application

1. Production is steady, so that results of past, present and future lots are broadly indicative of a continuing process.
2. Lots are submitted sequentially in the order of their production.
3. Inspection is by attributes, with the lot quality defined as the proportion defective.
4. Human involvement should be less in the manufacturing process.

E. OC Function

Associated with each sampling plan there is an OC curve, which portrays the performance of the sampling plan against good and poor quality. The probability that the lot will be accepted under a given sampling plan is denoted by $P_a(p)$ and a plot of $P_a(p)$ against given value of the lot quality p will give the OC curve.

OC function of the SSP is

$$P_a(p)=\sum_{X=0}^{c}\{[e^{(-np)}(np)^x]/X!\},$$

F. Determination of the Lot Size

By fixing the probability of acceptance $P_a(p)$ of the lot as 1-3.4 x 10^{-6} with Poisson Distribution as the basic distribution the value of np is obtained for c=0 and c=1 and finally the lot size is determined using the formula

$$AOQ=\frac{p.P_a(N-n)}{N}$$

provided by Duncan (1986) and are presented in Table 1 and Table 2 for various values of n and AOQ using Excel packages.

Example 1

For a given n=350 and AOQ=$0.95\times10^{(-8)}$, the value of the lot size can be obtained from table 1 as N=15868. The selected Single Sampling Plan (SSP) based on six sigma quality is N=15868, n=350, c=0. The OC curve for the suggested plan is presented in figure 1.

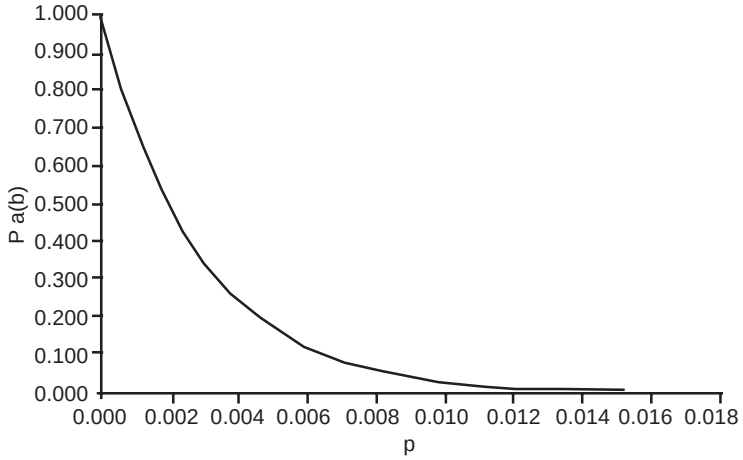
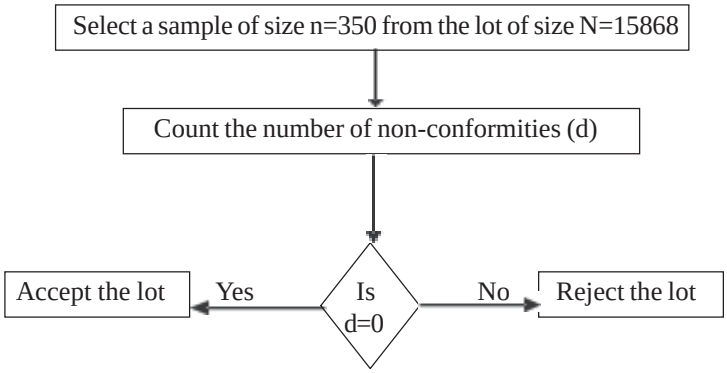


Figure 1 OC curve for SSP (n=350,c=0)

Explanation

In the manufacturing company if the consumer fixes AOQ as $0.95\times10^{(-8)}$ (95 non conformities out of 10 crores) and for a sample size 350 the lot size is 15868. In a lot of 15868 items select the sample of 350 items and count the number of non conformities. If the number of nonconformities is zero then accept the lot otherwise reject the lot and inform the management for corrective action. It is presented with following flowchart 2.

Flowchart 2



Example 2

For a given n=250 and AOQ=$1.03\times10^{(-5)}$, the value of the lot size can be obtained from table 2 as N=26009. The selected Single Sampling Plan (SSP) based on six sigma quality is N=26009, n=250, c= 1. The OC curve for the suggested plan is presented in figure 2.

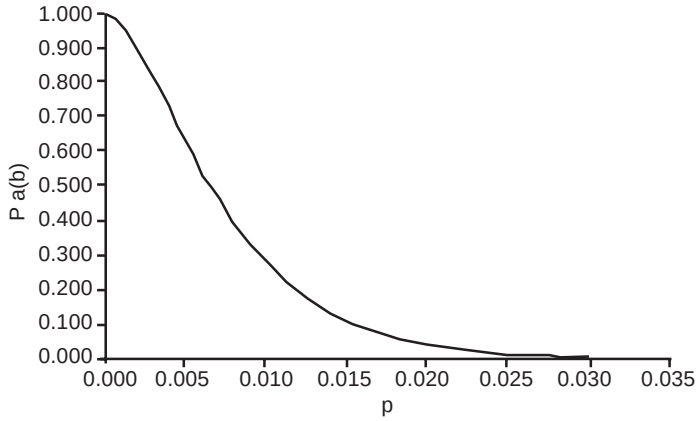


Figure 2 OC curve for SSP (n=250,c=1)

Explanation

In the pump manufacturing company if the consumer fixes AOQ as 1.03 x10 $^{(-5)}$, that is (103 non conformities out of 10 crores) and a sample size of 250 the lot size is 26009. In a lot of 26009 items select the sample of 250 items and if the number of nonconformities is less than or equal to one then accept the lot otherwise reject the lot and inform the management for corrective action.

Figure 3. Flowchart 3

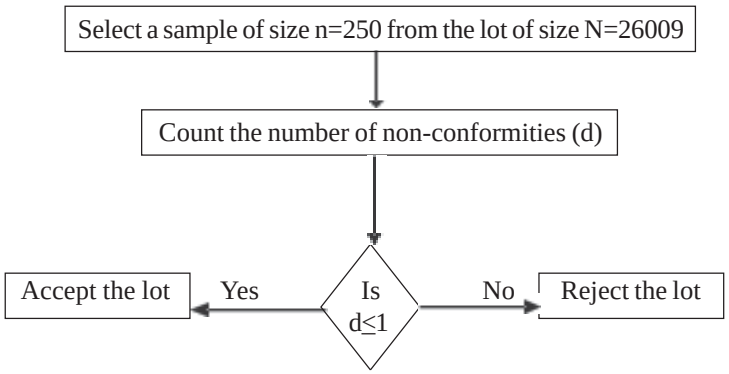


TABLE 1
VALUES OF THE LOT SIZE N FOR C=0, VARIOUS VALUES OF THE SAMPLE SIZE N AND AOQ

n / AOQ ($10^{(-8)}$)	500	450	400	350	300	250	200
0.10	586	519					
0.20	708	612	523				
0.35	1030	838	680	547			
0.40	1214	956	756	595			
0.45	1478	1113	850	652			
0.50	1889	1330	971	721	537		
0.55	2615	1654	1133	807	583		
0.60	4250	2186	1360	915	638		
0.65	11334	3221	1700	1058	703		
0.70		6120	2267	1253	785	515	
0.75		61214	3400	1535	887	557	
0.80			6800	1983	1020	607	
0.90				4760	1457	739	
0.95				**15868**	1855	829	
1.00					2550	944	
1.05					4080	1097	523
1.10					10201	1308	567
1.20						2125	680
1.25						3091	756
1.30						5667	850
1.35						34008	971
1.55							2267
1.60							3400
1.65							6800

TABLE 2
VALUES OF THE LOT SIZE N FOR C=1, VARIOUS VALUES OF THE SAMPLE SIZE N AND AOQ

n / AOQ($10^{(-5)}$)	500	450	400	350	300	250	200
0.10	619	544					
0.19	788	670	565				
0.39	2000	1385	1000	737	545		
0.47	5200	2412	1444	953	655		
0.49	8667	2962	1625	1028	690		
0.50	13001	3343	1733	1071	709		
0.51	26005	3836	1857	1117	729		
0.55		9361	2600	1348	821	531	
0.56		14627	2889	1422	848	542	
0.57		33437	3250	1504	876	553	
0.62			8667	2116	1054	619	
0.63			13001	2304	1099	634	
0.64			26006	2528	1147	650	
0.71				7914	1660	788	
0.72				11376	1773	813	
0.73				20226	1902	839	
0.74				91080	2053	867	
0.84					9751	1300	565
0.85					15603	1368	578
0.86					39017	1444	591
1.01						8668	897
1.02						13002	929
1.03						**26009**	963
1.17							2000
1.26							6501
1.27							8668
1.28							13003
1.29							26011

2. CONCLUSION

In this paper an attempt is made to determine the lot size of Six Sigma based single sampling Plans which has the probability of acceptance $P_a(p)$ of the lot as 1-3.4 x 10^{-6} using Poisson distribution as a base line distribution, acceptance numbers c=0 and c=1 and AOQ values for various values or a specified AOQ and sample size n. This will help the floor engineer to suggest on the size of the lot to be submitted for inspection for a fixed sample size and acceptance number. This work can be extended for other sampling plans having finite lot size. These plans are more suitable for the industries that are practicing Six Sigma methodologies.

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